

## Graphs in the Next Dimension

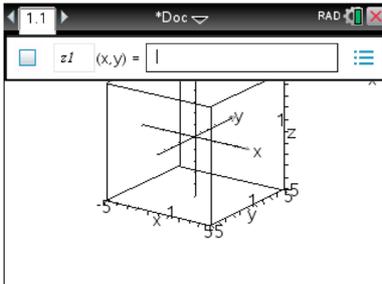
### Student Task Sheet

#### Introduction

The aim of this activity is to introduce you to 3D graphs, whilst reinforcing your understanding of some aspects of 2D graphs.

#### Task 1 - Familiarisation

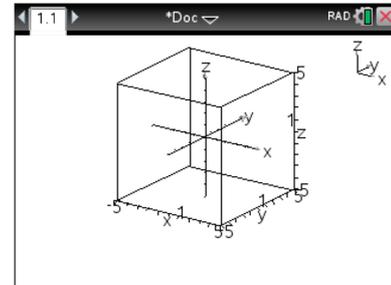
To start off with a blank 3D graphing template, do the following:



1. Start a **New Document** and select the **Graphs** application.
2. Press **[menu]** then select **View ... 3D Graphing** and you will obtain a page similar to that on the left.
3. Close the Graph Entry line by either pressing **[esc]** or pressing **[ctrl]** then **[G]**.

4. With your screen looking like the image on the right, you shall first examine what you have....

*Notice the small axes in the top right hand corner, and how they are orientated the same as the main graphing box in the centre of the screen.*



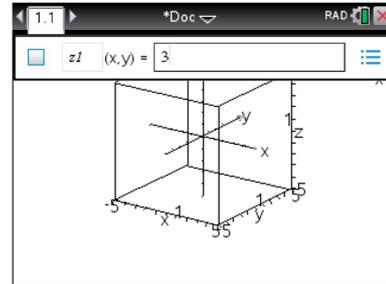
5. Now press any direction on the cursor keypad (**▲**, **▼**, **◀** or **▶**). You can also press and hold each direction.
6. Now press the letter **[O]**. Can you see what this did?
7. Now press the letter **[X]**. Again, what did this do?
8. Now press the letters **[Y]** then **[Z]**.  
**Write down** in your jotter, as precisely as you can, explanations for what views you see when you press each of **[X]**, **[Y]** or **[Z]**.
9. Now press **[O]** to return to the **O**riginal view.  
Press **[A]** to **A**nimate the graph. You can also press the cursor keys whilst it is spinning!  
You can stop the animation by pressing **[esc]**.

**Task 2 – Your First Plane**

1. Press **□** to return to the **Original** view.
2. Open the Graph Entry line by **either** pressing **tab**, **or** pressing **ctrl** then **G**.
3. Type in the number 3, so that  $z_1(x,y)=3$  and press **enter**.

4. You now see a plane whose z-coordinate is always 3. Press **X**, **Y**, **Z**, **□**, **A** and the arrow keys to view it from all directions.

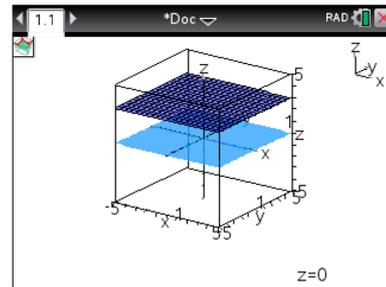
*Notice that whilst the view is rotating, the axes labels are re-displayed on the front edges of the graphing box.*



5. You will now introduce a z-trace to the graphing window.

Firstly, press **□** to return to the **Original** view. Press **menu** and select **Trace ... z Trace**. You will see another plane appear, with equation  $z=0$ .

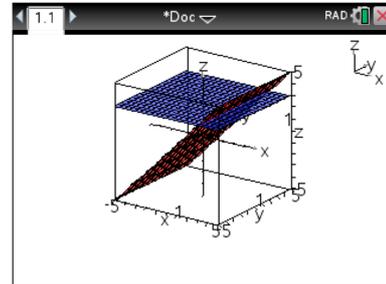
You can move this plane up and down by pressing and **holding down** **↑shift** whilst **also** pressing the cursor directions **▲**, **▼**, **◀** or **▶**.



You can exit z-trace mode by pressing **esc**.

### Task 3 – Slanting Planes

1. Press **2nd** to return to the **Original** view.
2. Open the Graph Entry line by **either** pressing **tab**, **or** pressing **ctrl** then **G**.
3. You will have the prompt for  $z_2(x,y)$  on show. Type in  $x$ , so that  $z_2(x,y)=x$  and press **enter**. You should see a screen like that shown on the right.



4. You now see a plane whose z-coordinate is controlled by whatever the x-coordinate is. Press **X**, **Y**, **Z**, **○**, **A** and the arrow keys to view it from all directions.

5. Press **2nd** to return to the **Original** view. Can you predict what will happen if  $z_2(x,y)=x$  becomes  $z_2(x,y)=2x$  ?

To check your prediction, edit  $z_2(x,y)$  so that it reads  $z_2(x,y)=2x$ .

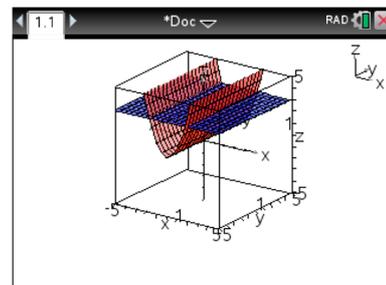
6. You can spend further time predicting and checking what  $z_2(x,y)=3x$  and  $z_2(x,y)=4x$  would look like.

7. You can also consider  $z_2(x,y)=-x$  and  $z_2(x,y)=-2x$ .

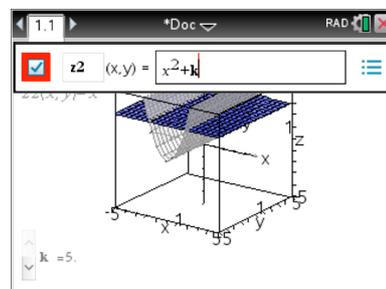
**Write down** any similarities and differences that you have noticed when you compare graphing in 2D and graphing in 3D and you change the *coefficient* of  $x$  to different values.

### Task 4 – Curving Planes

1. Press **2nd** to return to the **Original** view.
2. Edit  $z_2(x,y)$  so that it reads  $z_2(x,y)=x^2$  and press **enter**.
3. **Predict** what  $z_2(x,y)=x^2 - 3$  will look like. **Then** graph it.



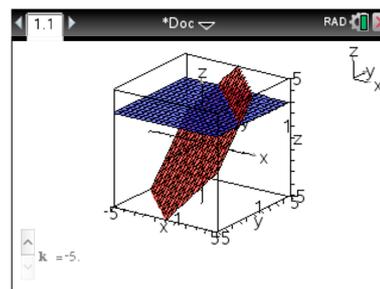
4. Consider  $z_2(x,y)=x^2 + k$  where  $k$  is an integer. You could **insert a minimised vertical slider**, naming the variable as **k**, place it in the bottom left corner of the screen and edit  $z_2$  to include the **k**.



**Write down why** changing the value of  $k$  has the effect that it does on the curved plane.

**Task 5 – Plane equations involving both x and y coordinates.**

1. Press  to return to the **Original** view.
2. Edit  $z2(x,y)$  so that it reads  $z2(x,y)=x+y$   
View it from all angles, and notice how it lies in relation to the axes and the  $z1$  plane.
3. **Write down** a reason why the  $z2$  plane appears to slant at an angle to the coordinate axes.
4. **Write down** a convincing reason for how you know whether the  $z2$  plane goes through the origin, or not.
5. Look at where the  $z1$  and  $z2$  planes intersect. How many intersection points are there? And what do all these intersection points combine to create?



**Write down** the (x, y, z) coordinates of one point that lies on both  $z1$  and  $z2$ .  
**Write down** the (x, y, z) coordinates of four more points that also lie on both  $z1$  and  $z2$ .  
 It is points like these that generate the intersection line of the two planes.

Now compare this 3D situation of two planes intersecting with the 2D graphing equivalent of two lines intersecting. **Write down** any similarities and/or differences between them.

6. Now consider very carefully the next plane equation:  

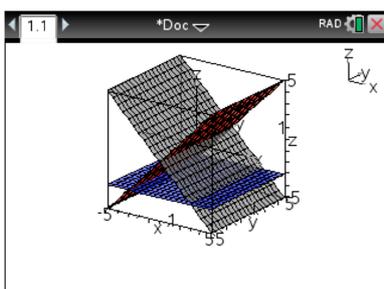
$$z2(x,y)= x^2 +y^2$$

**Before** you graph it on your TI-Nspire, sketch what you think it will look like.  
 Do spend time on this task – it is very satisfying to correctly predict a graph that is new to you.

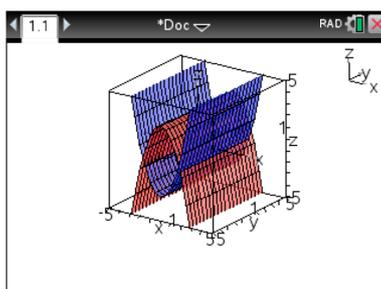
7. Then, in a similar fashion, predict what  $z2(x,y)= x^2 +y^2-5$  will look like, and then check it.

**Task 6 – More Challenging Plane Graphing.**

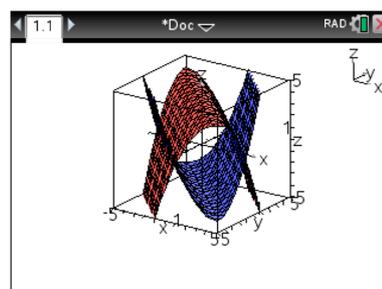
1. Can you discover the equations of the planes that are shown in the following 3 screenshots? In each, the viewpoint shown comes after pressing .



Planes Problem 1



Planes Problem 2



Planes Problem 3

2. Now, for a bit of interest, there are very many other, more complex functions that give rise to interesting graphs. You can try some of these on a new 3D graphing page:

$z1(x,y)= x^2 -y^2$

$z1(x,y)= 1/(x+y)$

$z1(x,y)= \text{sign}(x \cdot y)$

$z1(x,y)= \text{abs}(x+y)$

.... can you explain why this 'sign' graph looks the way that it does?

.... can you determine what the 'abs' function does?